Forced Convection Mass Transfer: Part II. Effect of Wires Located Near the Edge of the Laminar Boundary Layer on the Rate of Forced Convection from a Flat Plate

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The local and the average rates of forced convection through laminar boundary layers on a flat plate were shown to be markedly increased by locating small cylinders near the outer edge of the boundary layer. The local rate of forced convection was strongly peaked directly beneath each cylinder; the magnitude of the effect depended upon the free stream velocity, the spacing between cylinders, and the gap between the cylinders and the plate. Under optimum conditions, local values of the rate of forced convection were increased as much as 340%, while the average values were increased by over 190%.

In the first paper in this series (*1*) the rate of forced convection through a laminar boundary layer on a flat plate was shown to be substantially unaffected by free stream turbulence until the turbulence level, u'/U_o, exceeded a threshold value. For the particular conditions of that study, the rate of forced convection was in good agreement with the theoretical values (2) for zero free stream turbulence for $u'/U_o < 2.8\%$. For turbulence levels greater than 2.8%, the rate of forced convection increased in a regular fashion as u'/U_o increased and almost tripled at a turbulence level of 11%. In general it is not possible to control the turbulence level of a flowing fluid in order to attain these large turbulence levels except by placing some device in the stream itself.

An obstruction placed in a flowing fluid is often referred to as a turbulence promoter when it disturbs the flow in the vicinity of a surface in such a way that there is an increase in the rate of forced convection. Although the term *turbulence promoter* is associated with these devices, in most cases the mechanism responsible for the observed increase in forced convection is not known. In a few cases a regular recirculation pattern is observed (*3, 4*) that is more characteristic of laminar than turbulent flow. Almost invariably turbulence promoters cause a greater increase in pressure drop than in the rate of forced convection. In many cases the compensation for this disadvantage is an increase in design flexibility. Consequently, for design purposes it is necessary to have reliable data on both rate of forced convection and on pressure drop in order to arrive at the optimum configuration for any particular system.

Many different geometries have been proposed for use as turbulence promoters. These include protuberances attached to the transfer surface (*3, 5*), twisted inserts to cause a swirling motion to the fluid (*6*), and bluff objects located away from the surface (*7, 8*). The bluff body turbulence promoter studies reported in the literature were made with both streamlined and disk-shaped bodies of revolution located concentrically in cylindrical channels (*7, 8*). Flow was turbulent in all cases. The ratio of maximum bluff body diameter to channel diameter *d* covered a range of 0.4 to 0.875. The maximum increase in the average heat transfer coefficient of 400% was obtained with disks having a diameter ratio of 0.875 spaced two tube diameters apart (*8*). The increase in pressure drop corresponding to these conditions was 30,000% at the same mass flow rates (*8*). The pressure drop for streamlined shapes was generally ¼ to ½ that for the disks under the same conditions and the overall heat transfer coefficient was about 20 to 50% lower (*8*).

The present work is part of a program of studying the interaction of wakes from geometrical shapes located away from surfaces with the fluid in the boundary layer immediately adjacent to the surface. The particular configuration chosen for this study was a flat plate with small diameter cylinders located one-half to five cylinder diameters from the surface near the edge of the boundary layer. The discovery of conditions under which significant interactions occur could have important implications for the development of turbulence promoters. This is particularly true for applications in which the turbulence promoters are required to have the common characteristics of neither blocking the heat or mass transfer surface nor having stagnant regions in which solids can accumulate.

EQUIPMENT AND PROCEDURE

The local rate of forced convection through laminar boundary layers on a flat plate was determined by measuring the difference in level of a flat naphthalene plate before and after exposure to an air stream. The experiments were carried out in a once-through wind tunnel with a 6-ft long 3 × 12-in. working section made from 1-in. thick transparent plastic.

The naphthalene was contained in a 1/16-in. deep by 6 × 17-in. recess in a 11-15/16 × 18 × 3/16-in. flat stainless steel plate. The leading edge of the plate was sharpened to a knife edge flat on the upper side with 11 deg. included angle; the plate was supported by four strips of metal 16½-in. long, 7/32-in. wide, and ½-in. high. The surface level of the napthalene was measured to the nearest 0.0001 in.; it is estimated (*9, 10*) that less than 0.00005 in. of naphthalene sublimed during the surface measurement. The duration of a test run was chosen so that at least 0.002 and less than 0.015 in. of naphthalene sublimed. All runs were made at ambient temperature to minimize temperature variations along the naphthalene surface. The temperature depression of the mass transfer surface due to sublimation of the naphthalene was estimated to be always less than 0.2°C. based on the relation derived by Sogin (*10*), which was experimentally verified by Träss (*9*). Mean and maximum flow velocities were measured with a pitot tube, a calibrated vane type of anemometer, or with smoke tracers, depending on the velocity range used for a particular test.

The mass transfer Stanton number corresponding to the difference in surface level of the naphthalene caused by exposure of the flat plate in the wind tunnel was calculated from the expression

$$N_{St} = \frac{k_c}{U_o} = \frac{N_w \, P_{bar} \, M_A}{P_N \, \rho_A \, U_o \, M_N} \qquad (1)$$

and the Schmidt number for the naphthalene-air system ($N_{Sc} = 2.44$ at 25°C.) was calculated from the expression

$$N_{Sc} = 7(T_K)^{-0.185} \qquad (2)$$

recommended by Sherwood and Träss (*11*). Additional details on the equipment and procedure may be found in the first paper in this series (*1*).

The wires used as turbulence promoters in this study were hollow cylinders 0.093-in. O.D. and 0.070-in. I.D. and 11¾-in. long. Hollow cylinders were used to minimize the sag of the cylinders under their own weight. The cylinders were supported by special holders resting on the side edges of the stainless steel plate. The centerline of the cylinders could be located at distances of 1.6, 2.1, 3.2, and 5.4 cylinder diameters from the surface of the plate (gaps between the bottom of the cylinder and the plate of 1.1, 1.6, 2.7, and 4.9 cylinder diameters). The maximum change in the distance between the cylinders and the naphthalene surface due to naphthalene sublimation was 0.15 cylinder diameter, that is, ±5% at the closest spacing and ±1% for the greatest separation between cylinder and surface. Multiple cylinders could be located one behind the other at uniform center-to-center distances of 3.8, 6.5, 8.9, 10.8, 16.1, and 32.3 diameters.

The first cylindrical turbulence promoter was always placed at least 2 in. downstream from the leading edge. Therefore a portion of the data from each run provided an internal calibration giving an exact value of the rate of mass transfer in the absence of turbulence promoters for the particular conditions being studied. This permitted a more accurate evaluation of the maximum local value of the mass transfer coefficient and of the integrated average values achieved with any given promoter.

EXPERIMENTAL RESULTS

A pronounced increase in the rate of mass transfer through the laminar boundary layer was observed when an 0.093-in. diameter cylinder was located normal to the flow in a plane parallel to the mass transfer surface in the vicinity of the outer edge of the laminar boundary layer. The rate of mass transfer peaked very sharply directly underneath the cylinder and there was a pronounced dependence of the amount of increase on the distance of the cylinder from the plate. The area over which the cylinder caused an increase extended 2.5 to 3 diam. on either side of the cylinder, provided the bottom of the cylinder was within 2.5 diam. of the surface.

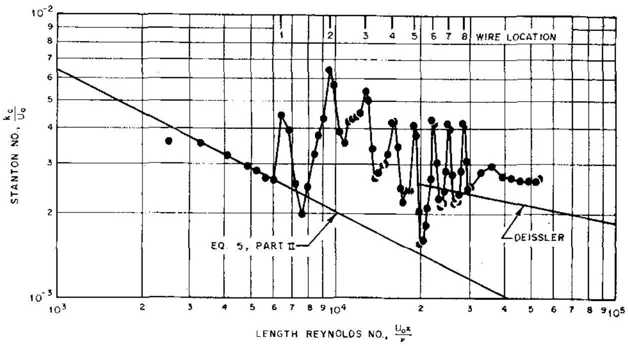

Fig. 1. Effect of 8 0.093-in.-diameter cylinders spaced 1 in. behind the other at a distance of 0.2 in. above the plate on the rate of mass transfer from a flat plate.

When multiple cylinders were placed one behind the other in a plane parallel to the mass transfer surface, a pronounced increase in the rate of mass transfer occurred beneath each cylinder. Typical results are shown in Figure 1 as local mass transfer Stanton number plotted against length Reynolds number. In the test shown in Figure 1 the free stream velocity was 6.4 ft./sec. and there were eight 0.093-in. cylinders spaced 10.8 cylinder diameters apart center-to-center and located with their centers 2.1 cylinder diameters above the mass transfer surface. Under these conditions at the location of the first cylinder, the thickness of the displacement boundary layer δ^* defined by

$$\delta^* = 1.729 \; x/\sqrt{(N_{Re})_x} \qquad (3)$$

was 0.58 cylinder diameter and the total boundary layer thickness δ defined by

$$\delta = 5.0 \; x/\sqrt{(N_{Re})_x} \qquad (4)$$

was 1.68 diameters.

To evaluate the increase in rate of forced convection caused by the presence of cylinders, it is necessary to possess accurate information on the rate in the absence of cylinders. Thus the line with a slope of − ½ in Figure 1 represents the rate of mass transfer through laminar boundary layers in the absence of turbulence promoters as described by Equations (16) and (18) of Part I (*1*)

$$(N_{St})_x = \frac{0.334\,[1 + 0.60\,(Tu - 2.8)^{1/2}]}{(N_{Sc})^{2/3}\,(N_{Re})_x^{1/2}} \qquad (5)$$

TABLE 1. RATIO OF PEAK STANTON NUMBER WITH PROMOTERS TO STANTON NUMBER IN THE ABSENCE OF PROMOTERS FOR DATA OF FIGURE 1

		Peak Stanton number ratio*		Cumulative increase in average mass transfer	
Cylinder No.	Length Reynolds No. × 10^{-4}	Laminar boundary layer	Turbulent boundary layer†	Laminar boundary layer	Turbulent boundary layer†
1	0.64	1.7	—	1.2	—
2	0.96	3.1	—	1.5	—
3	1.3	3.0	—	1.7	—
4	1.6	2.6	—	1.9	—
5	1.9	2.7	—	1.9	—
6	2.2	3.1	1.7	1.9	1.8
7	2.5	3.2	1.7	1.9	1.7
8	2.9	3.9	1.7	2.0	1.6

* Peak Stanton number ratio calculated from value at maximum and local value for either laminar or turbulent boundary layers in the absence of promoters.
† Transition assumed to occur at $(N_{Re})_x = 2.0 \times 10^4$.

for $Tu > 2.8\%$. The data on the left of Figure 1, obtained before any effect due to the cylinders, served to confirm the validity of Equation (5). The local rate of mass transfer through turbulent boundary layers was shown in Part I to be substantially independent of turbulence level for turbulence levels less than 7% and was satisfactorily described by the relation derived by Deissler (*12*). Thus those two curves labeled "Eq. 5, Part II" and "Deissler" represent base lines that permit accurate evaluation of the effect of turbulence promoters.

The ratio of peak local Stanton number beneath the cylinders to local Stanton numbers for the same condition in the absence of cylinders and the ratio of integrated Stanton numbers for the data in Figure 1 are given in Table 1. There is a possible ambiguity in the data of Table 1 because of premature transition from a laminar to a turbulent boundary layer caused by the cylinders [in the absence of cylinders the transition should occur at $(N_{Re})_x = 9 \times 10^4$ for the conditions of Figure 1]. Based on the pronounced minimum in the rate of mass transfer after the fifth cylinder, it is possible that transition occurred at $(N_{Re})_x = 2 \times 10^4$, and so values for both laminar and turbulent boundary layers are included in Table 1 for Reynolds numbers greater than this value. In either event the maximum local value of the Stanton number ratio was greater than 3.0 and the integrated average ratio of promoted to unpromoted Stanton number over the entire area beneath the cylinders was between 1.6 and 2.0.

Single Cylinder

Initial studies of the peaking of the rate of forced convection beneath cylinders showed that the maximum value was a complicated function of the space between cylinders, the space between the cylinder and the plate, and the free stream velocity. Therefore the majority of the studies were made with either one or two cylinders at various spacings and mean stream velocities. In all the subsequent discussions the Stanton numbers referred to will always be those corresponding to the maximum value observed on the surface beneath the cylinder.

Data for the ratio of the Stanton number at the peak (directly beneath the cylinder) to the Stanton number in the absence of promoters at the same location, $(N_{St})_{peak}/N_{St}$, are shown in Figure 2*A* for a single 0.093-in. diameter cylinder located 2 in. from the leading edge of the plate with distance of the cylinder from the plate and free stream velocity as the independent variables. Three scales are shown for the abscissa: the length Reynolds number corresponding to the location of the cylinder and the free stream velocity; the free stream velocity; and the frequency of vortex shedding calculated, assuming that the cylinder was located far from the channel walls.

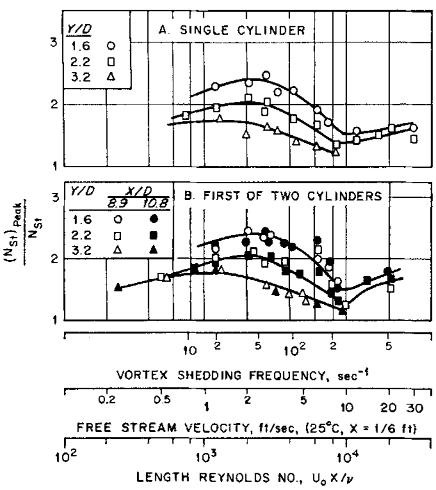

Fig. 2. Effect of a free stream velocity on peak value of mass transfer beneath a cylinder. (*A*) Single cylinder located 2 in. from leading edge of plate (*B*) First of two cylinders located 2 in. from leading edge of plate.

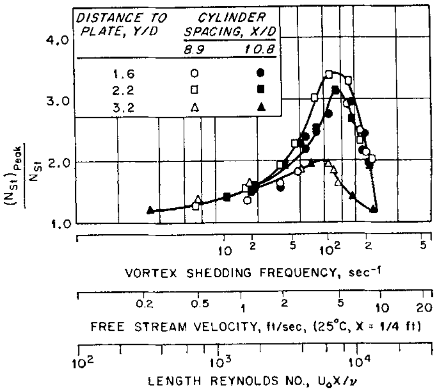

Fig. 3. Effect of free stream velocity on peak value of mass transfer beneath second of two cylinders. Cylinder located 3 in. from leading edge of plate.

The maximum increase in local rate of forced convection through laminar boundary layers obtained with a single cylinder was 240%. As can be seen from Figure 2*A*, this increase occurred when the free stream velocity was 2.5 ft./sec. [$(N_{Re})_x = 2.5 \times 10^3$] and when the cylinder center line was 1.6 cylinder diameters above the surface. For these particular conditions, the upper edge of the cylinder was located at the edge of the boundary layer calculated from Equation (4).

Two Cylinders

Values of the ratio $(N_{St})_{peak}/N_{St}$ for the first of two cylinders are shown in Figure 2*B*. The curved lines are identical with the curves from the single-wire data plotted in Figure 2*A*, thus permitting detection of any differences caused by the second cylinder. The maximum increase in the local rate of forced convection for the first of two cylinders occurred under the identical conditions observed with a single wire. The remainder of the data was in good agreement with the curves from Figure 2*A* except at a velocity of 6.5 ft./sec. [$(N_{Re})_x = 6.5 \times 10^3$], where there were indications of a second maximum in the local rate of forced convection with a value of $(N_{St})_{peak}/N_{St} = 2.30$.

The maximum increase in local Stanton number caused by the second cylinder was both substantially larger and more strongly dependent on velocity than was the maximum observed with the first or single cylinder. Comparison of Figures 2 and 3 shows that the maximum value of $(N_{St})_{peak}/N_{St}$ was 340% for the second of two cylinders compared to the value of 240% observed with a single cylinder. The second cylinder maximum occurred at a velocity of 5 ft./sec. or length Reynolds number of 7.5×10^3. In contrast to the behavior of the single cylinder data [in which it was not clear that the maximum value of $(N_{St})_{peak}/N_{St}$ had occurred even when the cylinder center was 1.6 diam. from the surface], the second cylinder maximum occurred when the cylinder center was located 2.2 diameters above the surface. In addition there was a small but real effect of the distance separating the cylinders on the maximum value of the peak Stanton number for the limited range covered by the two distances used for the data shown in Figure 3.

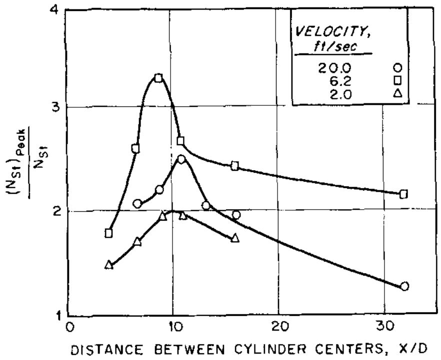

Fig. 4. Effect of distance between cylinder centers on peak Stanton number beneath second of two cylinders for cylinders with centers 2.2 diameters above the surface.

When the distance between cylinder centers was varied over a wide range, a pronounced effect on the second cylinder peak was observed as shown in Figure 4. These data were obtained with cylinders whose centers were located 2.2 diameters above the surface and with velocities of 20, 2.0, and 6.3 ft./sec. The latter velocity is close to the optimum of 4 to 6 ft./sec. shown in Figure 3. It is clear from Figure 4 that the spacing between cylinders to obtain the maximum peak value of the rate of forced convection beneath the second cylinder is a rather weak function of the velocity, although additional data are required to determine the quantitative relation.

Pressure Gradient

In Part 1 (*1*) the pressure gradient along the surface of the flat plate was measured with the use of a plate with the same dimensions as the mass transfer plate, except that there was no recess in the top surface and there were twenty-six 1/32-in. diameter pressure taps along the centerline of the plate. The variation in pressure along the surface of the plate was presented in terms of the pressure ratio

$$P_r = \frac{P_s - P_a}{P_t - P_a} \tag{6}$$

and the data from Part I in the absence of cylinders are shown as the crosshatched area in Figure 5. The small favorable pressure gradient corresponded to a value of $\Lambda' = 0.08$ where Λ' is given by

$$\Lambda' = [L/(\rho_A U_o^2/2g_c)]\, dP_s/dX \tag{7}$$

The new data shown on Figure 5 were obtained with two, four, and eight cylinders spaced 10.8 cylinder diameters apart, center to center, and located with their centers 2.2 cylinder diameters above the surface. The pressure readings were taken at locations between the cylinders rather than directly beneath them. Under these circumstances the value of Λ' calculated from Equation (7) by use of the data shown in Figure 5 increased from 0.08 in the absence of cylinders to 0.52 along the plate beneath the first four cylinders and 0.13 along the plate under the last four. When there were only four or two cylinders located above the plate, the value of Λ' was about 0.5 along the plate beneath the cylinders. Recovery of pressure occurred along the plate in the wake region behind the last cylinder. The value of the adverse pressure gradient was $\Lambda' = -0.13$.

DISCUSSION

Both the local and the average rates of forced convection mass transfer were markedly increased when small cylinders were placed in the vicinity of the edge of the laminar boundary layer on a flat plate. The maximum increase in the local rate of forced convection observed beneath a single cylinder or beneath the first of two cylinders was 240%. With multiple cylinders, the wake from upstream cylinders interacted with the flow field in the vicinity of downstream cylinders causing an additional increase in the local rate of forced convection over that observed with single cylinders. The maximum increase in the local rate of forced convection through laminar boundary layers on a flat plate beneath the second of two cylinders was over 340%. Although the local rate of forced convection was strongly peaked directly beneath the cylinder, the effect of the cylinder extended from 2.5 to 3.0 cylinder diameters on either side of the maximum. Consequently, the increase in the average rate of forced convection was as much as 190% over a considerable distance along the surface. (In one particular case an increase of 190% was caused by two cylinders 8.9 diam. apart located with their centers 2.2 diam. above the plate; the integration to determine the average rate of forced convection was carried out over a distance of 25 cylinder diameters along the plate.)

The initial studies showed that the observed increase in the local rate of forced convection was a complicated function of the free stream velocity and of both the distance between succesive cylinders and the distance between the cylinders and the plate. Although additional factors undoubtedly play an important role, obstruction of the area for flow by the cylinder with consequent increase in velocity was apparently not an important factor. In the present case the cylinders reduced the flow area by only 0.33%, resulting in only an insignificant increase in free stream velocity. In contrast to this, prior studies with bluff or streamlined promoters located away from the surface (*7, 8*) were made with bodies that blocked a substantial portion of the area, that is, from 16 to 76.5%, with an attendant increase in the velocity at the location of the obstruction of from 19 to 425%. Undoubtedly, this marked increase in velocity was primarily responsible for the maximum increase in friction loss of 30,000% reported in one prior study (*8*) of the effect of axial, bluff-body promoters on heat transfer and pressure drop inside a tube under turbulent flow conditions.

An upper limit on the increase in friction loss due to the presence of the cylinders in the present study may be obtained by calculating the total drag for a single cylinder and then assuming that each cylinder acts independently. For the conditions corresponding to the data in Figure 1, this procedure gives a value for the drag of the eight cylinders that is 16.6 times the drag on the flat plate in the absence of cylinders. However, as G. I. Taylor has pointed out (*13*), the resistance of several wires located one behind the other in the plans of flow is less than when

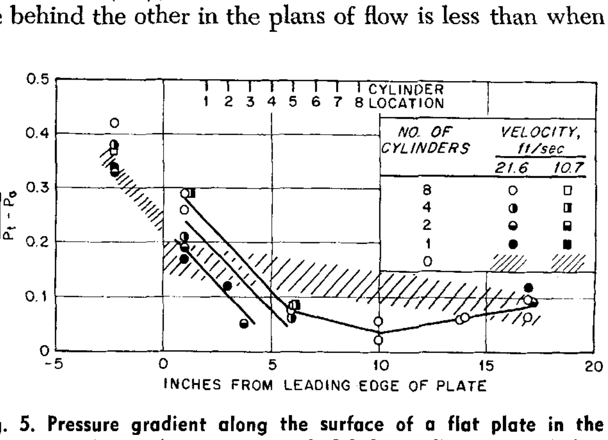

Fig. 5. Pressure gradient along the surface of a flat plate in the absence and in the presence of 0.093-in. diameter cylinders located 2.2 cylinder diameters from the surface.

the wires are in a plane normal to the flow. Consequently, the drag ratio of 16.6 calculated, assuming the cylinders act independently, represents a pronounced overestimate. A more accurate procedure would result from accounting for the relative contribution to the total drag of the skin friction and the form drag of the n cylinders in a sequence

$$(F_t)_n = n\alpha F_f + n\beta F_p \tag{8}$$

The major factor affecting the value of α, the coefficient of the skin friction term, is the reduced velocity in the wake behind the first cylinder. Data given by Kovasznay (*14*) show that the velocity on the centerline behind a cylinder is 60 to 80% of the free stream velocity for distances from 8 to 20 cylinder diameters from the cylinder. Since the skin friction force (*15*) is proportional to $U_o^{3/2}$, the value of α may be estimated as

$$\left(\frac{U_w}{U_o}\right)^{3n/2} < \alpha < \left(\frac{U_w}{U_o}\right)^{3/2} \tag{9}$$

or for the data of Figure 1, $0.07 < \alpha < 0.7$; the use of the geometric mean gives an estimate for the value of $\alpha \approx 0.2$. The value of β, the coefficient of the form drag term, is more difficult to estimate but a reasonable lower limit would appear to be $n\beta = 1$. Using these values, a more reliable estimate of the increased friction loss due to the cylinders is 2.3 times that of the plate in the absence of the cylinders for the data of Figure 1. The only data available to check Equation (8) and the estimated values of α and β are for two cylinders placed one behind the other at variable spacing (*18*). For distances separating cylinders of from 4 to 10 diam., the combined drag coefficient of the two cylinders was substantially constant at 1.50. Equation (8), with $\alpha = 0.2$ and $n\beta = 1$, gives a value of the drag coefficient of 1.57. Thus, Equation (8) may be somewhat conservative rather than representing a lower limit; however, much additional data are required to confirm this.

The principal factors responsible for the observed peaking of the local rate of forced convection beneath cylinders are believed to be an increase in the velocity gradient at the surface and the interaction of the cylinder wake with the fluid in the boundary layer. There are also indications that there may be secondary effects due to boundary layer instability at certain length Reynolds numbers (*16*) and effects due to vortex street interactions (*17*). Therefore, additional studies are required in order to develop reliable design procedures for cylindrical promoters located away from the transfer surface.

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NOTATION

c_p = specific heat, B.t.u./lb.$_m$
D = cylinder diameter, ft.
D_v = molecular diffusivity of mass, sq. ft./sec.
F = drag force, lb.$_f$/sq. ft.
g_c = conversion factor, (lb.$_m$/lb.$_f$) (ft./sec.^{2})
k = thermal conductivity, B.t.u./(sq. ft.) (sec.)/(°F.) (ft.)
k_c = mass transfer coefficient, ft./sec.
L = total length, ft.
M = molecular weight, lb.$_m$/mole
n = number of cylinders, dimensionless
N_{Pr} = Prandtl number, $c_p\mu/k$, dimensionless
N_{Re} = Reynolds number, xU/ν, dimensionless
N_{Sc} = Schmidt number, ν/D_v, dimensionless
N_{St} = Stanton number, k_c/U_o, dimensionless
N_w = mass flux, lb.$_m$/sq. ft.-sec.
P = pressure, lb.$_f$/sq. ft.
Tu = turbulence level (%), dimensionless
T_K = temperature, °Kelvin
u' = fluctuating velocity, ft./sec.
U_o = free stream velocity, ft./sec.
U_w = wake velocity, ft./sec.
X = length, ft.
Y = distance to plate, ft.

Greek Letters

α,β = coefficient, dimensionless
δ,δ^* = thickness, ft.
ν = kinematic viscosity, sq. ft./sec.
ρ = density, lb.$_m$/cu. ft.
μ = viscosity, lb.$_m$/ft.-sec.
Λ' = form factor, dimensionless

Subscripts

A = air
a = atmosphere
f = skin friction
N = naphthalene
n = number of cylinders
o = free stream
p = form drag
r = reduced
s = static
t = total
x = length

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